

## Coarse-grained molecular models of water: a review

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(Received 23 January 2012; final version received 27 February 2012)

Coarse-grained (CG) models have proven to be very effective tools in the study of phenomena or systems involving large time- and length-scales. By decreasing the degrees of freedom in the system and using *softer* interactions than seen in atomistic models, larger time steps can be used and much longer simulation times can be studied. CG simulations are widely used to study systems of biological importance that are beyond the reach of atomistic simulation, necessitating a computationally efficient and accurate CG model for water. In this review, we discuss the methods used for developing CG water models and the relative advantages and disadvantages of the resulting models. In general, CG water models differ with regard to how many waters each CG group or bead represents, whether analytical or tabular potentials have been used to describe the interactions, and how the model incorporates electrostatic interactions. How the models are parameterised, which typically depends on their application, is also discussed.

**Keywords:** molecular simulation; anomalies; mapping; force field development; coarse graining



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### 1. Introduction

Water is, most likely, the most utilised solvent in experimental and computational studies [1–3], particularly for biological systems. However, water has many unique properties, such as a density maximum at 277 K and increased diffusivity upon compression, which complicates its dynamics and physics compared to other solvents [4]. Because of its importance as a

solvent in biological systems and its unique properties, developing an accurate computational model for water is an ever-evolving and ongoing quest.

In biological systems and biological processes, the time- and length-scales involved typically prevent the use of fully atomistic simulations because of the computational cost and coarse-grained (CG) models are instead used to overcome atomistic limitations. In CG models, groups of

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atoms are represented by single sites or beads to reduce the number of degrees of freedom in the simulations and allow for larger time steps (i.e.,  $\sim 10$  to  $20$  fs, rather than  $1-2$  fs as is commonly used in atomistic simulations) so molecular dynamics simulations can be run for very long times.

In some cases, a system's solvent, including water, is modelled implicitly (see for example [5–7]). Essentially, the CG model of the solute is optimised to interact as it would in the presence of a solvent, without explicitly modelling the solvent molecules. While implicit models are computationally efficient, they can easily miss important physical details. For example, although hydrophobic solutes typically have little effect on the solvent with changing concentration, ionic solutes like NaCl affect how the solvent behaves at higher concentrations [8].

While the techniques and practices behind using implicit solvents are interesting, with this review we have chosen to focus on explicit CG models for water, where the emphasis is on the development of accurate models with respect to water's properties and behaviours rather than the solute (as is the case with implicit solvent models). In particular, we aim to introduce the reader to the most recent and the most popular CG water models developed for use in molecular dynamics or Monte Carlo simulations. First, in Section 2, we summarise the main coarse-graining methods used to develop CG water models, and also models for lipids, proteins and polymers, as well as highlighting water-specific cases. In Section 3, we review water models where one water is mapped to each bead followed by multi-water models in Section 4. Finally, in Section 5, we discuss models with analytical potential functions with and without explicit electrostatics.

## 2. Coarse-graining methods

Although there are many CG methods available with which to develop accurate force fields for solvents and solutes of interest, they all typically share common procedures. The first step involves 'mapping' the molecule from the atomistic level to the CG level. Before the mapping begins, the degree of coarse-graining must be determined; in other words, how many and which atoms should be assigned to each bead. Traditionally, no more than six heavy atoms (non-hydrogen atoms) are represented by a single CG bead, because too much detail is lost with higher levels of coarse-graining and isotropic potentials become inadequate with respect to representing the beads. The other major component of mapping is retaining specific functional groups, such as the choline group of certain phospholipids.

In molecules with multiple heavy atoms (e.g., lipids), mapping specific atoms from atomistic simulations to pre-determined sites is somewhat trivial and only requires simple intuition. The difficulty with water is determining

which waters should be mapped to which beads since the water molecules move independently and are only bound by non-bonded interactions [9]. Because of this, three strategies are currently used to map water to the CG level. The first, and perhaps the simplest method, is to map one water molecule to one CG bead. This strategy is usually implemented with centre-of-mass (COM)-based techniques, as discussed below, but only saves a marginal amount of simulation time. More commonly, water is implicitly mapped (i.e., each bead is assigned a fixed number of arbitrary water molecules rather than explicitly mapping atomistic waters to the CG level) and beads interact with each other through an analytical potential; the CG bead is meant to represent multiple waters as dictated by its size, interaction strength, and other thermodynamic and structural properties. Finally, as detailed in Section 4, clustering algorithms can be used as a way to dynamically determine which waters belong to which beads in COM-based CG methods [9,10].

With a mapping strategy determined, the next step is to define the force field used to dictate the thermodynamics and physics of the CG model. In short, the input parameters of the force field must be optimised in a way to reproduce macroscopic properties of interest (i.e., experimental or simulated surface tension, structure, diffusivity and density data). Unfortunately, the loss of detail prevents matching all properties, with some behaviours being mutually exclusive in relation to fitting. For example, as shown by Izvekov and Voth [11], diffusivity and radial distribution functions (RDFs) cannot be matched simultaneously. Likewise, Toth found that total energy and structure could not be fitted simultaneously [12]. As such, the key to this step is deciding what properties need to be retained on the CG level, which, in turn, usually determines the most appropriate coarse-graining method. For example, if the goal of the CG model is to study structural behaviour the most appropriate CG method is one that fits the interaction potentials to RDFs, but a different method would be more appropriate if the target were accurate free energies of solvation or dynamical properties.

There are two major types of targets for fitting the CG force field: experimental measurements and measurements taken from atomistic simulations. When atomistic simulations are used as the target, the CG method is usually a COM-based method, where the mapping determines which atoms at the atomistic level contribute each bead and the COM of the atoms within a bead is then used to determine the position of the bead on the CG level. When experimental measurements serve as the optimisation target, the mapping only serves as a guide to interpret atomistic level information from the CG simulations rather than to enable atomistic data to be extracted to the CG level.

With the mapping and target for the CG model defined, what remains is to parameterise the CG force field. If a

numerical (or tabulated), rather than an analytical, potential is to be developed, there are three main approaches, all of which are COM-based CG methods: force-matching (FM), developed by Voth and co-workers [11,13–17], inverse Monte Carlo (typically depicted as IMC), first reported by Lyubartsev and coworkers [18–21], and the iterative Boltzmann inversion (IBI) method initially developed by Reith et al. [22]. In contrast, no general methods have been reported to parameterise analytical expressions for CG force fields fitted to target experimental properties. Rather, each CG model developed either uses some kind of iterative approach to achieve the optimisation they desire or a fitting procedure unique to their CG model. As such, the methods used to develop water models with an analytical expression will be discussed separately in the subsequent sections.

In the FM approach, the potential is iterated until the forces between the CG molecules match the forces between their atomistic counterparts [11,15–17]. In short, a target atomistic molecular dynamics simulation is run and a set of reference forces are collected to which the set of CG forces are subsequently matched by minimising Equation (1)

$$\chi^2 = \frac{1}{3LN} \sum_{l=1}^L \sum_{i=1}^N |F_{il}^{\text{ref}} - F_{il}^p(g_1, \dots, g_m)|^2, \quad (1)$$

where  $N$  and  $L$  are the total number of atoms and atomic configurations, respectively, and are the parameters in the force field (e.g., the site diameter of a CG bead) where,  $1 \leq i \leq m$ . In cases where the CG model is to be used for Monte Carlo simulation instead of molecular dynamics, Toth developed an energy-matching algorithm analogous to the FM strategy [12].

The IMC method [18–21] works by fitting input parameters to observables taken from the target atomistic simulation using Equation (2),

$$\Delta \langle A \rangle^k = \langle A \rangle^k - A^* = \sum_{\gamma} \frac{\partial \langle A \rangle^k}{\partial \alpha_{\gamma}^k} \Delta \alpha_{\gamma}^k + O\left(\left(\Delta \alpha_{\gamma}^k\right)^2\right), \quad (2)$$

where  $A^*$  is the target observable,  $\alpha_{\gamma}^k$  represents the input parameters, and  $\langle A \rangle^k$  the CG ensemble average of the observable. From the definition of an ensemble average, the value of the gradient of the observables can be estimated via Equation (3)

$$\frac{\partial \langle A \rangle^k}{\partial \alpha_{\gamma}^k} = -\beta \left( \left\langle A^k \frac{\partial H}{\partial \alpha_{\gamma}^k} \right\rangle - \langle A^k \rangle \left\langle \frac{\partial H}{\partial \alpha_{\gamma}^k} \right\rangle \right), \quad (3)$$

where  $H$  is the total Hamiltonian of the system and  $\beta$  is equal to  $1/kT$ , where  $k$  is Boltzmann constant and  $T$  is the simulation temperature. By combining Equations (2) and (3), the required change in the parameters can be calculated to update the force field in an iterative fashion until the CG observables converge to the target values. An appealing feature of this method is the built-in sensitivity

analysis with respect to the observable/input parameter gradient, which can indicate which parameters have the most influence on the observables in question.

Finally, the IBI method works by iteratively optimising the CG potential until radial distribution functions measured in the CG simulation match those from the target atomistic simulation mapped to the CG level using Equation (4),

$$V_{i+1}(r) = V_i(r) + \delta kT \ln \frac{g_i(r)}{g^*(r)}. \quad (4)$$

In Equation (4), the tabulated potential  $V_i(r)$  is updated at each point,  $r$ , by comparing the target RDF  $g^*(r)$  with the CG RDF,  $g_i(r)$ . Also shown in Equation (4) is a damping factor,  $\delta$ , that was not included in the original work as the authors were able to reach convergence without one; however, several groups [9,23–30] have found adding a damping factor with values less than 1.0 is necessary in ill-defined optimisation schemes. Specifically, Hadley and McCabe encountered a diverging oscillatory response when optimising many different (coupled) CG potentials simultaneously or for systems in a well-structured (crystalline) state [27–29].

While the above three methods are the most common in the literature, many other novel and application-specific methods for deriving tabulated CG potentials exist. For example, Wei and Wang [31] developed a novel way to retain a large degree of atomistic detail in systems solvated by water, but accelerate dynamical processes so that coarse-graining is not even necessary. Their damped short-ranged interaction (DSRI) smooths out the potential energy surface by subtracting out a short ranged correction term,

$$U_{sr}^{LJ} = \left\{ 4\epsilon \left[ \left( \frac{\sigma}{r} \right)^{12} - \left( \frac{\sigma}{r} \right)^6 - \left( \frac{\sigma}{r_{sr}} \right)^6 + 6 \left( \left( \frac{r}{r_{sr}} - 1.0 \right) \right) \right. \right. \\ \left. \left. \times \left( 2 \left( \frac{\sigma}{r_{sr}} \right)^{12} - \left( \frac{\sigma}{r_{sr}} \right)^6 \right) \right], \quad r \leq r_{sr} \right\} \\ U_{sr}^{\text{coul}} = \left\{ \frac{q_i q_j}{4\pi\epsilon_0} \left( \frac{1}{r} + \frac{r}{r_{sr}^2} - \frac{2}{r_{sr}} \right), \quad r \leq r_{sr} \right\}, \quad (5)$$

scaled by  $\lambda$  (0.3) from the original Hamiltonian at distances less than 0.40 nm ( $r_{sr}$ ). In equation (5),  $\epsilon$  represents the interaction energy,  $\sigma$  the hard sphere diameter,  $r_{sr}$  the short-ranged cut off,  $q_i$  the charge on atom  $i$ , and  $\epsilon_0$  the permittivity of a vacuum. With these values, the RDF of pure water is still reproduced, but the diffusion coefficient is drastically increased due to less friction between molecules and softer interactions. The authors considered this change in diffusivity their goal as they wanted to minimise the barrier to diffusion to accelerate processes like self-assembly. Using this methodology, a CG model for dimyristoyl-phosphatidylcholine was shown to self-assemble within 40 ns from a completely random initial configuration, after which,  $\lambda$  can

be set to 0.0 and the simulations run without the correction term [31].

### 3. Single water models

As mentioned previously, CG water models with a one-water mapping limit the amount of time saved by using a CG model. However, with COM-based techniques, mapping multiple waters to single beads for force field development is difficult and has only recently been overcome [9,10]. As such, CG water models from FM [11,15,16], IMC [19,32], and IBI [23,33,34] have traditionally been limited to mapping one water to each bead.

While the structuring and dynamics obtained from simulations from these water models are generally good, they do require longer simulation times than other, more CG water models. Additionally, the work of Wang et al. [34] provides interesting insight into accuracy issues and the isotropic nature of the interactions in single water CG models. In their work, the IBI method was used to fit RDFs for single water beads based on the SPC [35], SPC/E [36] and TIP3P [37] atomistic water models and found accurate RDFs were mutually exclusive to accurate tetrahedral packing; if RDFs were fitted within line thickness, the tetrahedral packing was much lower than the target, and if the model was modified to fit tetrahedral packing, the output RDF was much more structured. The authors attributed this to the isotropic nature of a CG model derived from the IBI method. Without the directionality of hydrogen-bonding between beads, they could not maintain the tetrahedral organisation found in water, and with tetrahedral configuring enforced, the water hydrogen-bonded in an isotropic fashion causing a much higher degree of structure.

In the rest of this section, we review single water CG models that treat water in a novel way, or significantly reduce the simulation times whilst still retaining a level of atomistic detail comparable to COM-based CG water models. Borgis and coworkers [38–40] developed a single site model for water called the polarisable pseudo-particle (PPP) model. The PPP model retains accurate hydrodynamics and solvation, whilst minimising simulation time and speed up of diffusion for similar reasons as stated for the DSRI method [31] discussed above. Originally [38], the model used a 12-6 Lennard-Jones (LJ) potential,

$$V_{\text{LJ}}(r) = 4\varepsilon_{ij} \left[ \left( \frac{\sigma_{ij}}{r} \right)^{12} - \left( \frac{\sigma_{ij}}{r} \right)^6 \right], \quad (6)$$

with  $\varepsilon_{ij}$  equal to the interaction strength and  $\sigma_{ij}$  equal to the diameter between sites  $i$  and  $j$ , but many properties, including density and vaporisation enthalpy, were inaccurate. To overcome these problems, Masella et al. [40] refined the original potential by adding a sum of

Gaussian functions,

$$U_{\text{pp}}^{\text{corr}} = \sum_{i=1}^N \sum_{j>i}^N \sum_{k=1}^3 \varepsilon_{pp}^k \exp \left( \frac{-(r_{ij} - r_k)^2}{\gamma_k} \right), \quad (7)$$

where  $\varepsilon$  is an energy term,  $r_k$  is the Gaussian centre, and  $\gamma_k$  is the width of the Gaussian peak and a density energy term (used to maintain the local solvent density near a particle),

$$n_i^k = \sum_{i=1, i \neq j}^N f_k(r_{ij}), \quad (8)$$

$$f_0(r_{ij}) = \phi_0(r_{ij}), f_{k>0}(r_{ij}) = \phi_k(r_{ij}) - \phi_{k-1}(r_{ij}), \quad (9)$$

$$\phi_k(r_{ij}) = \begin{cases} 1 & \text{if } r_{ij} < R_{\min} \\ \frac{P_5((r_{ij} - R_{\min}))}{(R_{\max}^k - R_{\min}^k)} & \text{if } R_{\min} \leq r_{ij} \leq R_{\max}^k \\ 0 & \text{if } r_{ij} > R_{\max}^k \end{cases}, \quad (10)$$

$$U_{\text{pp}}^{\text{density}} = \sum_{i=1}^N u_{\text{pp}}^{\text{density}}(i) = \sum_{i=1}^N \varepsilon_s^0 (n_i^0 - \bar{n}^0)^2 + \varepsilon_s^1 (n_i^1 - \bar{n}^1)^2. \quad (11)$$

In Equations (8) through (11),  $R_{\min}$  corresponds to the first peak of the water RDF,  $R_{\max}$  corresponds to the location of the minimums in the RDF,  $P_5$  is a fifth order switching function, and  $\bar{n}$  represents the average number of neighbouring particles.  $\varepsilon_s^0$  and  $\varepsilon_s^1$  were fitted to heat of vaporisation and the free energy required to create a cavity in the solvent. The revised PPP model was shown to not only reproduce better pure solvent properties, but also still retain the ability to provide good structuring of biomolecules like polypeptides.

Vlcek and Nezbeda introduced a *primitive model* for water aimed at simplifying calculations, whilst retaining hydrogen-bonding features [41,42]. To achieve this, an attractive square-well potential was implemented between hydrogen-bonding partners (oxygen and hydrogen in water) and a hard sphere potential was used for every other pair-wise interaction. The square-well depth, attractive cut-off, and the hard sphere diameters were derived from fully atomistic models (SPC/E, TIP4P and TIP5P) using perturbation theory. The authors were able to show, despite an absence of long-ranged interactions, that these simple models quantitatively describe the phase behaviour and structural properties of water in Monte Carlo simulations.

In a similar attempt to capture hydrogen bonding in a simple manner, Dill and co-workers constructed the Mercedes-Benz water model [43], so named because water is represented as a two-dimensional disk with three symmetrical hydrogen-bonding vectors stemming from the centre similar to the Mercedes-Benz<sup>®</sup> logo. The centres interact through a 12-6 LJ potential and through a Gaussian-based orientation dependent hydrogen-bonding potential,

where equal but opposite vectors from different discs have the strongest attraction. A number of the thermodynamic anomalies found within water (such as the density maximum at lower temperatures and large anomalous heat capacity) could be simulated with this model, but its use is limited considering its two-dimensional nature. As a result, Bizjak et al. extended the model to a three-dimensional form with four tetrahedral hydrogen-bonding vectors to enhance its accuracy and realism [44,45].

One issue with both the primitive and Mercedes-Benz models is their dependence on using Monte Carlo versus molecular dynamics simulations. As such, dynamic properties and phenomena cannot be studied, no matter how accurate or computationally efficient. In order to enforce tetrahedrality in a model compatible with molecular dynamics, Molinero and Moore introduced the so-called monotonic water (mW) model adapted from the Stlinger–Weber silicon potential [46,47]. In the mW model the water molecules interact through a LJ Gaussian potential and a three body tetrahedral potential, described by,

$$E = \sum_i \sum_{j>i} \phi_2(r_{ij}) + \sum_i \sum_j \sum_k \phi_3(r_{ij}, r_{ik}, \theta_{ijk}),$$

$$\phi_2(r_{ij}) = A\varepsilon \left[ B \left( \frac{\sigma}{r_{ij}} \right)^4 - 1 \right] \exp \left( \frac{\gamma\sigma}{r_{ij} - a\sigma} \right), \quad (12)$$

$$\phi_3(r_{ij}, r_{ik}, \theta_{ijk}) = \lambda\varepsilon [\cos \theta_{ijk} - \cos \theta_o]^2 \times \exp \left( \frac{\gamma\sigma}{r_{ij} - a\sigma} \right) \exp \left( \frac{\gamma\sigma}{r_{ik} - a\sigma} \right),$$

where  $r_{ij}$  is the distance between particles  $i$  and  $j$ ,  $\theta$  is the angle between three waters, and  $\lambda$  scales the repulsive tetrahedral term,  $\phi_3$ . The tetrahedrality,  $\lambda$ , is fit to vaporisation enthalpy, the interaction energy,  $\varepsilon$ , is fitted to the experimental melting temperature, and the site diameter,  $\sigma$ , is fitted to the density of water at room temperature. The remaining model parameters in Equation (12) are taken from the original silicon model [46]. While this model is not very coarse-grained, its major source of timesaving stems from the lack of electrostatics and the small non-bonded interaction cut-off of 0.432 nm, which is large enough to reproduce the structure of water.

In a further study, the mW model was combined with a CG model of DNA [48] in which the other CG beads interacted with water via the potential from Equation (12) with the tetrahedrality parameter equal to zero. The DNA model parameters were fitted to reproduce RDFs, the number of neighbours in the first solvation shell seen in atomistic simulations, and the residence time for solvation. The mW model proved to be very compatible with this CG model in a molecular dynamics framework (as opposed to strictly Monte Carlo). Molinero and co-workers also found that the mW model could reproduce the structure and

anomalies in liquid and solid water, enabling the study of ice [49] and clathrate hydrate formation [50,51].

Instead of using exotic geometries to reproduce water's anomalies, de Oliveira et al. [52,53] added a Gaussian term to a 12-6 LJ potential as per Equation (13),

$$V_{LJ}(r) = 4\varepsilon_{ij} \left[ \left( \frac{\sigma_{ij}}{r} \right)^{12} - \left( \frac{\sigma_{ij}}{r} \right)^6 \right] + a\varepsilon \exp \left[ -\frac{1}{c^2} \left( \frac{r-r_0}{\sigma} \right) \right], \quad (13)$$

and found that a single water bead could reproduce thermodynamic, dynamic, and structural anomalies in water due to the introduction of a soft shoulder in the potential. Chaimovich and Shell fitted the parameters in Equation (14) to minimise the relative entropy,

$$S_{\text{rel}} = \sum_i p_{AA}(i) \cdot \ln \left( \frac{p_{AA}(i)}{p_{CG}(i)} \right), \quad (14)$$

where  $p$  is the probability of configuration  $i$  [54] in the all atom (AA) and coarse-grained (CG) models. In essence, when Equation (14) is minimised, the configurations that the CG model encounters should be identical to the configurations encountered by the atomistic counterpart. While the anomalies and RDFs of water could be reproduced through this optimisation scheme, the anomalies are only accurate near the state conditions of the density maximum.

When the goal of a study is to ensure the reproduction of the unique anomalies found within water, single water models appear to be most appropriate as they incorporate the necessary features to retain a high level of accuracy whilst minimising simulation time. If these anomalies are of little interest to the simulation in question (i.e., in biological simulations), a multiple water CG model may be a better choice.

#### 4. COM-based multiple water models

Until only recently, multiple water models compatible with COM-based coarse-graining methods have been missing from the literature. To the best of our knowledge, the work by Hadley and McCabe was the first to successfully map multiple waters to a single CG bead compatible with a COM-based approach [9]. The motivation behind the work was to develop a computationally efficient water model compatible with the IBI method [22]. Because an atomistic RDF mapped to the CG level is used as the target of the optimisation in the IBI method, a method to identify the different clusters of waters to be mapped to a CG bead as a function of time is needed. In the work of Hadley and McCabe [9] the  $k$ -means cluster optimisation algorithm [55,56] was used. The  $k$ -means algorithm identifies groupings of data points, which when applied to the mapping of multiple atomistic waters to single CG beads, the data points represent the

water's spatial coordinates and the groupings represent the location of the mapped CG bead. Initially, the number of clusters,  $k$ , must be chosen and their starting coordinates decided. Normally, deciding the value of  $k$  is a major difficulty in proper application of the  $k$ -means algorithm, so there exist steps in variations of the algorithm to dynamically change  $k$  during the optimisation. A graphical representation of a generic application of the algorithm can be seen in Figure 1, where the squares represent waters, the spheres represent the location of the CG beads, the 'clouds' represent the allocation of waters to each cluster, and the arrows represent updating of the cluster's coordinates.

In the case of mapping water,  $k$  represents the number of CG beads, and, therefore, the average amount of waters mapped to each CG bead. In the development of the CG water model, this was varied so 1, 3, 4, 5, 6, 8, and 9 waters were mapped to each bead (called H2OX, where X equals the number of mapped waters), respectively, to assess the appropriate level of coarse-graining with the best balance between accuracy and computational efficiency. The starting locations of the clusters are chosen randomly from the coordinates of the oxygen atoms in the first frame of the trajectory and the final locations determined from the previous frame ( $i - 1$ ) served as the starting locations for subsequent frames ( $i$ ). The authors found that the target RDF had minimal variation with different starting locations in the first frame. In the next step, each data point (coordinates of the water molecule) is allocated to the nearest cluster (employing periodic boundary conditions) and the cluster's coordinates are then updated by finding the COM of each allocated water. With the new coordinates, the allocation is repeated iteratively until the termination criteria,

$$\sum_{n=1}^k (r_n^{i+1} - r_n^i)^2 < \text{tol}, \quad (15)$$

is satisfied. In Equation (5),  $r_n^i$  is the position of bead  $n$  at iteration  $i$  and  $\text{tol}$  is the tolerance set by the user (0.001 nm in [9]).

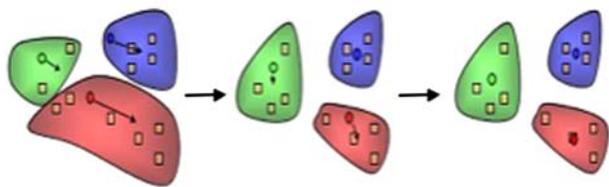


Figure 1. (Colour online) Schematic illustration of the application of the  $k$ -means algorithm to the clustering of water molecules. Circles represent the COM of a given cluster, squares represent the COM of water molecules, and shaded regions represent the allocation of waters to each cluster in a colour-coded fashion. Reprinted with permission from *Journal of Physical Chemistry B* [9].

Through the analysis of several different quantities Hadley and McCabe found, H2O4 to have the best balance between accuracy and computational efficiency. Additional insight was gained by comparing H2O4 to H2O1 (the one water mapping), in that H2O4 was found to be approximately 15 times faster than H2O1 in pure water simulations, which highlights the computational time involved in modelling single water beads in a CG simulation, and the RDF between water beads in the mixed system was actually better for the multiple water models than the single water model (H2O1). This was attributed to hydrogen-bonding between beads and within beads similar to the analysis by Wang et al. [34]. Between single water beads, hydrogen bonding has a large contribution to the interaction energy, but, in a multi-water bead, the majority of hydrogen bonding occurs within the bead and the hydrogen bonding across beads is negligible compared to the other non-bonded interactions between the beads as a whole. In other words, because hydrogen bonding is directional, an isotropic potential is less accurate in a single water model where hydrogen bonding has such a large contribution. Furthermore, the authors were able to show in a later publication [29] that the water model developed enabled good structuring of a lipid solute (hexadecanoic acid) and was capable of driving self-assembly between hexadecanoic acid and cholesterol.

A second multiple water mapping method called CUMULUS was recently reported by van Hoof et al. [10]. The motivation behind CUMULUS stems from the desire to map a specific number of waters to each bead, as opposed to an average number of waters as found in the H2O4 model [9]. In addition, the methodology can map ions to the water clusters in a controlled fashion. The CUMULUS method defines a 'division energy' through Equation (16),

$$E_{\text{div}} = \sum_i^Q \sum_{j=1}^{N_i} \sum_{k=1}^{L_i} \frac{r_{ij}^2 - \alpha r_{0i}^2}{\alpha r_{0i}^2}, \quad (16)$$

where  $Q$  equals the number of bead types,  $N_i$  the number of groups of bead type  $i$ ,  $L_i$  the number of molecules or atoms in bead type  $i$ , and  $\alpha$  is a shifting parameter in order to determine the mapping in each frame of the trajectory.  $r_{0,i}$  is a user-defined threshold for the root mean squared distance to the COM in a CG bead, which, basically controls the sphericity of a bead and the maximum distance between a heavy atom and its mapped site. By minimising the division energy, the CUMULUS model employs explicit four-water mapping (or one ion solvated by three waters) and maximises the sphericity of the CG beads. Using a Monte Carlo framework and simulated annealing to achieve a minimum in the division energy, CUMULUS can be used to determine which waters belong to which beads. The Monte Carlo moves allow the creation of a new bead from unmapped atoms, abandoning an old bead to unmap atoms, switch a free atom to an existent bead, or

switch atoms between beads. While this method can restrict the number of heavy atoms to a CG site, by definition of the Monte Carlo moves, some heavy atoms will not be mapped to the CG level. As such, technically each water bead will represent a little more than four waters per bead.

With the mapping from CUMULUS, target RDFs can be measured and fitted to by way of the IBI method. The authors mapped four water beads, three water beads with an ion, and a model for octanol to test their methodology and show how the model was transferable over a range of salt and octanol concentrations and could accurately reproduce the structure in the systems studied.

Although these are the only reports of algorithms that can be used for mapping multiple waters in a COM-based approach, given the advantages of such models we anticipate their ideas and concepts will be refined and built upon in future publications.

### 5. Analytical multi-water CG models

Without a doubt, the most used CG water model in the literature is the one developed by Marrink et al. in the MARTINI force field [57–59]. In the MARTINI force field, CG beads interact through a 12-6 LJ potential, with the addition of a Coulombic term for charged beads,

$$V_c(r) = \frac{q_i q_j}{4\pi\epsilon_0\epsilon_r r} \quad (17)$$

In Equation (17),  $q_i$  is the charge of bead,  $i$ ,  $\epsilon_0$  is the permittivity of a vacuum, and the relative dielectric constant  $\epsilon_r = 15$  for explicit screening.

In general, CG beads in the MARTINI force field are categorised with respect to their interaction type (i.e., polar, non-polar, etc.) and the interaction strength is taken from an interaction matrix [59]. The diameter of most MARTINI beads is set to 0.47 nm, while the interaction strength is fitted to free energies of vaporization, hydration, and partitioning. The MARTINI water model is described as a single charge-neutral bead (type P<sub>4</sub>) mapped to four waters. Due to the use of a 12-6 LJ potential, the high interaction strength, and the lack of explicit charges, the MARTINI water model has poor representation of many important properties (i.e., interfacial tension and compressibility) and actually requires anti-freeze particles [59], to prevent the water beads from solidifying as the water model can freeze at temperatures as high as 300 K under the right simulation conditions [59].

Attempts have been made by several authors to develop an improved water model within the MARTINI framework, where water does not freeze near room temperature. For example, Chiu et al. used a Morse potential,

$$V_M(r) = \epsilon \left[ e^{\alpha(1-(r/R_0))} - 2e^{1/2\alpha(1-(r/R_0))} \right], \quad (18)$$

instead of a LJ potential because of its softer nature [60]. The three model parameters ( $\epsilon$ ,  $a$ ,  $R_0$ ) were fitted to enthalpy of vaporisation, density, and surface tension data. While slightly more expensive than a LJ calculation, a much larger time step can be used due to the increased softness of the potential. In a similar work, Shinoda et al. used a 12-4 Mie potential instead of the 12-6 LJ or Morse potentials [61]. By fitting parameters to density and surface tension, instead of solvation free energies, the water model developed was found to describe the temperature dependence of these properties well but sacrifices structural agreement. The authors investigated many different forms of Mie potentials, but found 12-4 allowed for the best isothermal compressibility. While the 12-4 Mie potential was fitted for all water cross-interactions, a 9-6 Mie potential was used for all the solute-solute interactions. In a later publication, Shinoda et al. found that the model could accurately simulate bilayer bending, buckling and budding, because it was fitted to surface tensions as opposed to free energies [62].

To address issues of charge neutrality (i.e., the high relative dielectric permittivity and inaccuracies associated with charged sites), a polarisable version of the MARTINI water model was developed by Yesylevskyy et al. [63]. As shown in Figure 2, three sites represent this polarisable model; WP and WM are charged sites at fixed bond lengths from the centre site (W), but only interact via Coulombic interactions, while the centre site interacts only through the LJ potential. Because of the explicit charges involved, the relative dielectric constant can be lowered to a value of 2.5 from the original value of 15. Five variables in this model ( $\theta$ ,  $K_\theta$ ,  $q$ , bond length, and interaction strength of W) can be fitted to target properties, though the bond length and  $\theta$  were fixed at 0.14 nm and 0°, respectively, with the other three being altered systematically until both density and the dielectric constant matched experimental values. With these alterations, the model performs better than the original MARTINI water model with respect to reproducing the air/water surface tension and the freezing temperature [63]. In addition, the revised model allows for accurate free energy of ion transport across bilayers and the electroporation process of a bilayer matches the expected behaviour [63]. The major issue of the model is the higher computational expense due to the increased degrees of

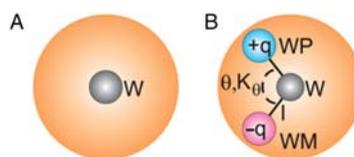


Figure 2. Schematics of the (A) original and (B) polarisable MARTINI water models. The shaded area indicates the van der Waals radii of the central particle W. Reproduced with permission from *PLoS Computational Biology* [63].

freedom and the inclusion of more electrostatic calculations. As such, the authors suggest reverting to the original model when dielectric screening is not an issue [63].

Wu et al. [64] also attempted to incorporate electrostatics into a CG water model compatible with the MARTINI force field through the use of a multipole in a four-water bead designed to match the properties of a 4-water cluster as closely as possible. The model, referred to as the big multipole water (BMW) model, is similar to the polarisable model of Yesylevskyy et al. [63] in that, it uses three sites to represent four waters, as shown in Figure 3, where the outer sites only interact through electrostatic forces. However, because the bonds and angles are kept rigid, the model is technically not polarisable. The geometry and charges of the BMW model were optimised to best reproduce both the dipole and the quadrupole moment of a four-water cluster in a COM mapping style. In the target atomistic simulation, a cluster is defined as a water molecule and its three nearest neighbours; the dipole moment and quadrupole moment of that cluster are measured in order to determine the target values on the CG level. In addition to electrostatic forces, the central sites interact through a modified form of the Born–Mayer–Huggins potential (Equation (19)), which is much softer than the LJ potential used within the MARTINI force field.

$$V(r) = \frac{\varepsilon}{1 - (f/a) - ((6-f)/12)} \left\{ \frac{6-f}{12} \left( \frac{r_m}{r} \right)^{12} - \left( \frac{r_m}{r} \right)^6 + \frac{f}{a} \exp \left[ a \left( 1 - \frac{r}{r_m} \right) \right] \right\}. \quad (19)$$

In Equation (19),  $\varepsilon$  and  $r_m$  are the depth and position of the minimum, respectively, while  $f$  and  $a$  determine the softness of repulsion and shallowness near the minimum. Like MARTINI, a relative dielectric screening must be used, which is fitted to experimental density alongside the four parameters in Equation (4) to experimental values for the interfacial tension, interfacial potential at the air–water interface, permittivity in bulk and isothermal compressi-

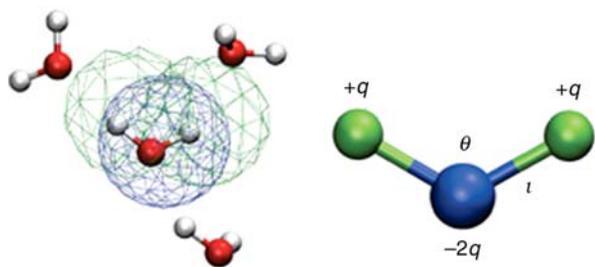


Figure 3. (Colour online) The 4-water clustering (left) described by the BMW model (right). The green beads interact through coulombic forces, while the centre bead (blue) interacts through both coulombic and van der Waals forces. Reproduced with permission from the *Journal of Physical Chemistry B* [64].

bility. Due to the fact these properties are fitted, BMW performs much better than other MARTINI-based water models, though it is computationally slower. When combined with the MARTINI lipid force field, BMW gives the correct dipole potential of a DPPC bilayer derived without any further optimisation. However, it should be noted that the potential was parameterised at 300 K and so it is not recommended for use at very different state conditions. Comparing these models, Bennett and Tieleman found both polarisable MARTINI water and BMW performed similarly in the application of studying pore formation in a lipid membrane [65]. While neither improves structure compared to the original neutral water model, the polarisable MARTINI water model does slightly improve the free energy.

Outside of the MARTINI framework for analytical multi-water CG models, the works of Darré et al. [66] and Riniker and van Gunsteren [67] have tried to extend the degree of coarse-graining of water compared to all the previously mentioned models, while still incorporating electrostatics and polarisability. Darré et al. proposed the WAT4 model in which four tetrahedral bound beads (Figure 4) collectively represent 11 waters [66]. Fitting the mass of WAT4 to the experimental room temperature density and dividing the final result by the molecular weight of one water molecule determined the degree of coarse-graining. Each bead has a partial charge fitted to dielectric

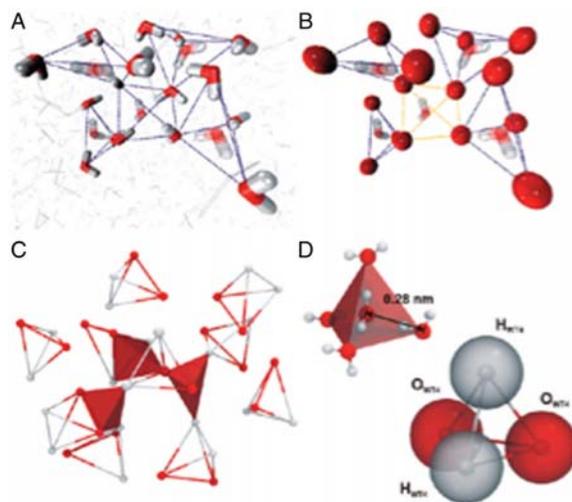


Figure 4. (Colour online) From atomistic to CG water. (A) Snapshot taken from a MD simulation showing the tetrahedral hydrogen-bonding network of water with hydrogen bonds indicated by dashed lines. (B) The location of CG beads in the model with explicit CG bonds shown as dark dashed lines and non-bonded interactions shown as transparent dashed lines. (C) Structural organisation of WT4 in the bulk solution taken from a MD snapshot. The shaded planes highlight the tetrahedrality between WT4 groups. (D) A single WT4 group in which the light beads represent the positively charged groups and the dark beads represent the negatively charged groups. Reproduced with permission from the *Journal of Chemical Theory and Computation* [66].

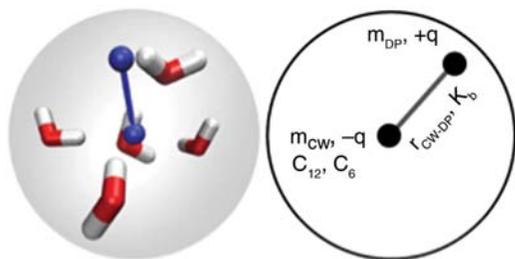


Figure 5. (Colour online) Schematic representation of the two-site CG water model representing five water molecules proposed by Riniker and van Gunsteren [67]. Reproduced with permission from the *Journal of Chemical Physics* [67].

permittivity, and the bonds between beads are fairly flexible to allow for polarisability and to make the structure more malleable. Because of its tetrahedral topology, WAT4 has a good solvation structure and was shown to solvate DNA well compared to a fully atomistic rendition.

Riniker and van Gunsteren developed a two-site water model designed to represent five atomistic waters [67]. The atomistic properties to be matched and compared against were tracked using a clustering algorithm similar to *k*-means [55,56], CUMULUS [10] or the BMW model [64], as discussed earlier. Each water and its four nearest neighbours belong to a separate cluster, but clusters that share waters with another cluster are deleted. To overcome reallocation problems stemming from self-diffusion of individual waters, waters in a cluster are loosely bound through a highly flexible ghost bond. As seen in Figure 5, the central site interacts through a 12-6 LJ potential and Coulomb's law, while the dipole particle (near the edge) only interacts through electrostatic forces. The mass of the central particle is twice the mass of the dipole particle to obtain the correct rotation of the dipole. The parameters were fitted to density and relative static dielectric permittivity. The bond was controlled through a harmonic oscillator so the model is polarisable. The model performed better than both BMW and MARTINI with respect to unfitted properties like compressibility or surface tension, does not require anti-freeze particles, and is more computationally efficient.

## 6. Conclusions

While atomistic level models for water have high accuracy and provide a large degree of detail, they can be computationally costly. With the softer interaction and reduced degrees of freedom found in CG models, they allow for longer time- and length-scales to be studied because of their computational savings. Phenomena appropriate for CG modelling typically uses water as a solvent, so the CG model of water must be accurate at the CG level. In addition, a large amount of water is typically

applied as a solvent, so CG models with maximum accuracy are of the greatest utility.

In this review, many degrees of coarse-graining in the development of CG water models have been discussed, including models with explicit electrostatics and polarisability for accurate treatment of CG ions, and exotic models to reproduce water's thermodynamic anomalies near their respective state conditions. These models are fitted to many different properties of water including surface tension, RDFs, density, and dielectric permittivity according to what properties are of interest for the CG model to accurately capture. While some CG models are only compatible within their own framework, others are designed to be compatible with CG models of other molecules of interest like lipids or DNA. Most of these developments have been within the last three years, so the field of coarse-graining water is still in its early stages and continues to evolve to enhance accuracy whilst retaining computational efficiency. Our hope for this review is to introduce readers to what has been developed and inspire them to take the models to the next stage of evolution.

## Acknowledgements

The project described was partially supported by grant number R01 AR057886-01 from the National Institute of Arthritis and Musculoskeletal and Skin Diseases. Resources were also provided by the National Energy Research Scientific Computing Center, supported by the Office of Science of the Department of Energy under contract number DE-AC02-05CH11231, and the National Science Foundation through TeraGrid grant number MCB080117N.

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